Exercise 71

a. Graph the functions f(x) = x/2 and g(x) = 1 + (4/x) together to identify the values of x for which

$$\frac{x}{2} > 1 + \frac{4}{x}.$$

b. Confirm your findings in part (a) algebraically.

Solution

The aim is to find the values of x for which f(x) > g(x).

$$f(x) > g(x)$$

$$\frac{x}{2} > 1 + \frac{4}{x}$$

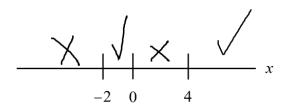
$$\frac{x}{2} - \frac{4}{x} - 1 > 0$$

$$\frac{x^2}{2x} - \frac{8}{2x} - \frac{2x}{2x} > 0$$

$$\frac{x^2 - 2x - 8}{2x} > 0$$

$$\frac{(x - 4)(x + 2)}{2x} > 0$$
(1)

The critical points are x = 4, x = -2, and x = 0. Partition the number line at these values of x and test a number within each interval to see if inequality (1) is true or false.



Therefore, the values of x for which f(x) > g(x) are

$${x \mid -2 < x < 0, \ x > 4}.$$

This is confirmed in the graph of the two functions versus x.

